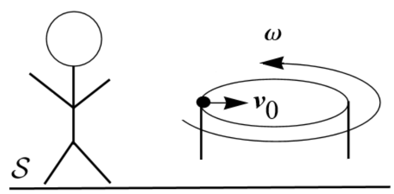
Problem 1.46

Consider the experiment of Problem 1.27, in which a frictionless puck is slid straight across a rotating turntable through the center O. **(a)** Write down the polar coordinates r, ϕ of the puck as functions of time, as measured in the inertial frame $\mathcal{S}$ of an observer on the ground. (Assume that the puck was launched along the axis $\phi = 0$ at $t = 0$.) **(b)** Now write down the polar coordinates r', ϕ' of the puck as measured by an observer (frame $\mathcal{S}'$) at rest on the turntable. (Choose these coordinates so that ϕ and ϕ' coincide at $t = 0$.) Describe and sketch the path seen by this second observer. Is the frame $\mathcal{S}'$ inertial?

Solution

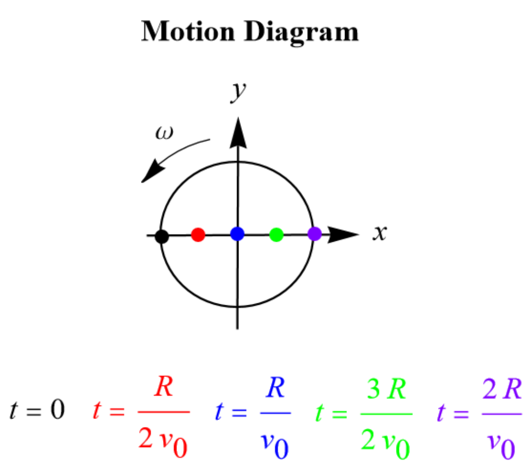
Part (a)

This answer applies for an observer standing off of the turntable, which has radius R.



Suppose the puck is launched from the left end with speed v_0 towards the center of the turntable. Since the puck is frictionless, the spinning of the turntable does not exert any force on the puck. By Newton's first law, then, the puck moves with constant velocity from the observer's point of view. The first law holds because $\mathcal{S}$ is an inertial frame (the floor the observer is standing on neither accelerates nor rotates).

$$\begin{cases} v_x = v_0 \quad \rightarrow \quad \dfrac{dx}{dt} = v_0 \quad \rightarrow \quad x = v_0 t + x_0 \quad \rightarrow \quad x = v_0 t - R \\[2ex] v_y = 0 \quad \rightarrow \quad \dfrac{dy}{dt} = 0 \quad \rightarrow \quad y = y_0 \quad \rightarrow \quad y = 0 \end{cases}$$



Change to polar coordinates (r, ϕ) by substituting $r\cos\phi$ for x and $r\sin\phi$ for y.

$$\begin{cases} r\cos\phi = v_0t - R \\ \\ r\sin\phi = 0 \end{cases}$$

To get r, square both sides and add the respective sides together.

$$r^2\cos^2\phi + r^2\sin^2\phi = (v_0t - R)^2 + (0)^2$$

Simplify both sides.

$$r^2 = (v_0t - R)^2$$

Take the square root of both sides.

$$r = |v_0t - R|$$

To get ϕ, divide the respective sides of the equations to eliminate r.

$$\frac{r\sin\phi}{r\cos\phi} = \frac{0}{v_0t - R}$$

Simplify both sides.

$$\tan\phi = 0$$

Therefore,

$$r(t) = \begin{cases} R - v_0t & \text{if } 0 \leq t \leq \frac{R}{v_0} \\ \\ v_0t - R & \text{if } \frac{R}{v_0} \leq t \leq \frac{2R}{v_0} \end{cases} \qquad \text{and} \qquad \phi(t) = \begin{cases} \pi & \text{if } 0 \leq t < \frac{R}{v_0} \\ \\ 0 & \text{if } \frac{R}{v_0} < t \leq \frac{2R}{v_0} \end{cases}.$$

Part (b)

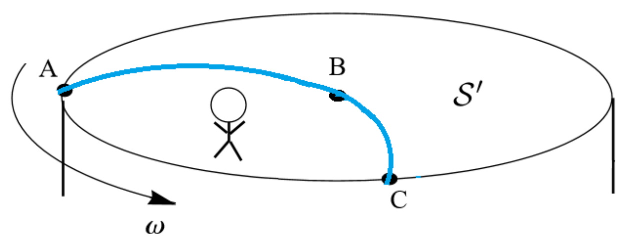
This answer applies for an observer standing on the turntable, which has radius R. Since the puck is frictionless, the rotation of the turntable exerts no force on it. This means that when the turntable rotates counterclockwise by an angle α, the puck's position rotates in the opposite direction by an angle α to the observer. Also, because there's no friction, the speed in the radial direction never changes. When the puck travels inward it has a negative radial velocity, and when the puck travels outward it has a positive radial velocity.

$$\begin{cases} \dfrac{dr'}{dt} = \pm v_0 & \rightarrow \quad r' = \pm v_0 t + r'_0 \\[2ex] \dfrac{d\phi'}{dt} = -\omega & \rightarrow \quad \phi' = -\omega t + \phi'_0 \end{cases}$$

When the puck passes through the origin at time R/v_0, it loses π radians instantly. In addition, at $t = 0$ and $t = 2R/v_0$, the puck is at the edge ($r' = R$). Therefore,

$$r'(t) = \begin{cases} R - v_0 t & \text{if } 0 \le t \le \frac{R}{v_0} \\[2ex] v_0 t - R & \text{if } \frac{R}{v_0} \le t \le \frac{2R}{v_0} \end{cases} \qquad \text{and} \qquad \phi'(t) = \begin{cases} \pi - \omega t & \text{if } 0 \le t < \frac{R}{v_0} \\[2ex] -\omega t & \text{if } \frac{R}{v_0} < t \le \frac{2R}{v_0} \end{cases}.$$

Below is a sketch of the frictionless puck's path as seen by the observer. A, B, and C are the start, middle, and end of the puck's path on the turntable, respectively.



The frame $\mathcal{S}'$ is not inertial because it's rotating.